NAIC-ID(RS)T-0227-95

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by

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HUMAN TRANSLATION

NAIC-ID(RS)T-0227-95 10 October 1995

MICROFICHE NR: 95C000634

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English pages: 32

Source: Unknown; pp. 1-12

Country of origin: China
Translated by: SCITRAN
F33657-84-D-0165
Requester: NAIC/TASC/Richard A. Peden, Jr.
Approved for public release: distribution unlimited.

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ABSTRACT: This article studies the problems of fixed time rendezvous between two spacecraft in general coplanar elliptical orbit under the effects of three horizontal impulses. Necessary conditions for completing rendezvous are derived, and methods are given for calculating the magnitudes of the three horizontal impulses as well as their times of application.

SUBJECT TERMS: Rendezvous Spacecraft Elliptic orbit Impulse

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I. INTRODUCTION

Following along with the development of manned space flight orbital work and the construction in space of large scale basic space facilities, people have more and more need for frequently applied rendezvous and docking technology. Beginning in the 1950's, there were already many scholars studying orbit transformation and space rendezvous problems [1-4]. However, most research work was limited simply to ideal circular target orbits. Otherwise, it would have created relatively large guidance errors. However, the orbits of target spacecraft (for example, malfunctioning spacecraft) are not necessarily circular orbits. Also, there is no need to definitely have to take target orbits and first turn them into circles. To this end, research should be done on rendezvous problems associated with elliptical orbits under general conditions. However, up to the present time, articles studying the utilization of horizontal impulses in elliptical orbits in order to complete spacecraft rendezvous missions have been few.

In another area, during considerations of the real problems of space flight engineering, the attitudes of many spacecraft are pointed toward the earth [5,6]. If, when orbits change, the directions of thrusts applied are not along horizontal directions at the places in question, then, the attitudes of spacecraft each time orbits change depart from normal flight attitudes, turning in a certain

* Numbers in margins indicate foreign pagination. Commas in numbers indicate decimals.

direction. Due to the fact that, during rendezvous processes, instances of orbital change are numerous, as a result, this will create very heavy pressures on spacecraft inertial guidance systems and ground monitoring and control. Moreover, there is a possibility of flight craft showing the appearance of the loss of flight status in association with earth objectives. If orbit change thrust directions are along horizontal directions at the places in question, it is then possible to resolve the series of problems discussed above. Therefore, research on using horizontal thrusts at the locations in question to carry out elliptical orbit optimal rendezvous is extremely necessary. This also holds the possibility of turning into a key path for the development of rendezvous technology. This article studies the problems of fixed time rendezvous of two spacecraft in general coplanar elliptical orbit under the influence of horizontal impulses.

During this research, it is assumed that target flight craft are moving in orbit IV right along. The initial orbit of the tracking flight craft is orbit I. After application of the first impulse, the tracking flight craft goes into orbit II. After moving for a time in orbit II, the second impulse is applied causing it to enter orbit III. Orbit III must intersect with orbit IV where the target flight craft is located. The tracking flight craft moves in orbit III. In conjunction with that, it arrives simultaneously with the target flight craft at this intersection point. At this time, a third impulse is applied to the tracking flight craft, causing it to have the same speed as the target flight craft and, thereby, completing rendezvous. See Fig.1.

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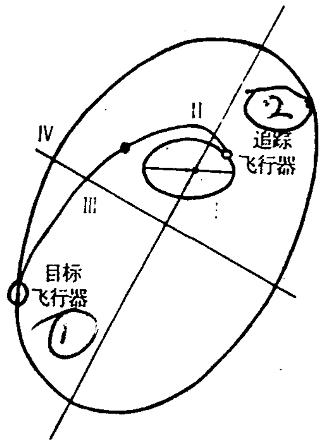

Fig.1 Multiple Impulse Rendezvous Schematic
Key: (1) Target Flight Craft (2) Tracking Flight Craft

Explanation of Symbols

hm, am, and em represent, respectively, target flight craft orbit (orbit IV) moments of momentum, half major axes, and eccentricities.

τm stands for the moments when target flight craft pass through perigee points.

rm1, fm1, and Em1 respectively stand for radius vectors, true anomalies, and eccentric anomalies associated with the target flight craft at the instant of the first impulse effect.

rm2, fm2, and Em2 respectively stand for radius vectors, true anomalies, and eccentric anomalies associated with the target flight craft at the instant of the second impulse effect.

rm3, fm3, and Em3 respectively stand for radius vectors, true anomalies, and eccentric anomalies associated

with the target flight craft at the moment of rendezvous (that is, the instant of the third impulse effect).

αm stands for the included angle between orbit I and orbit IV perigee lines of apsides.

h1, a1, and e1 respectively stand for orbit I moments of momentum, half major axes, and eccentricities.

h2, a2, and e2 respectively stand for orbit II moments of momentum, half major axes, and eccentricities.

h3, a3, and e3 respectively stand for orbit III moments of momentum, half major axes, and eccentricities.

r1, r2, and r3 respectively stand for radius vectors associated with tracking flight craft at the moments of the effects of the first, second, and third impulses.

f1 and E1 respectively stand for true anomalies and eccentric anomalies associated with tracking flight craft at the instant of the effects of the first impulse in orbit I.

f21 and E21 respectively stand for true anomalies and eccentric anomalies associated with tracking flight craft at the instant of the effects of the first impulse in orbit II.

f22 and E22 respectively stand for true anomalies and eccentric anomalies associated with tracking flight craft at the instant of the effects of the second impulse in orbit II.

f31 and E31 respectively stand for true anomalies and eccentric anomalies associated with tracking flight craft at the instant of the effects of the second impulse in orbit III.

f32 and E32 respectively stand for true anomalies and eccentric anomalies associated with tracking flight craft at the instant of the effects of the third impulse in orbit III.

$\tau 1$, $\tau 2$, and $\tau 3$ respectively stand for the instants when tracking flight craft pass through perigee points on orbits I, II, and III.

$\alpha 1$ and $\alpha 2$ respectively stand for the included angles between perigee lines of apsides for orbit I and orbit II and for orbit I and orbit III.

$\theta 1$, $\theta 2$, and $\theta 3$ respectively stand for the included angles between target flight craft radius vectors and tracking flight craft radius vectors at the instants of the effects of the first, second, and third impulses. Those in line with the direction of movement of the target flight craft are selected as positive values.

(x,y) and (Vx,Vy) respectively stand for the relative positions and relative speeds of two flight craft in a target orbit coordinate system (components in directions $\vec{i}$, and $\vec{j}$).

$\Delta V1$, $\Delta V2$, and $\Delta V3$ respectively stand for the magnitude of first, second, and third impulses.

$\Delta f1$ stands for the geocentric angle tracking flight craft turn through from the instant of the effect of the first impulse to the instant of the effect of the second impulse.

$\Delta f2$ stands for the geocentric angle tracking flight craft turn through from the instant of the effect of the second impulse to the instant of the effect of the third impulse.

$\Delta fm1$ stands for the geocentric angle target flight craft turn through from the instant of the effect of the first impulse to the instant of the effect of the second impulse. /3

$\Delta fm2$ stands for the geocentric angle target flight craft turn through from the instant of the effects of the second impulse to the instant of the effects of the third impulse.

Δf stands for the geocentric angle tracking flight craft turn through during the process of rendezvous.

Δfm stands for the geocentric angle target flight craft turn through during the process of rendezvous.

μ stands for the earth's gravitational constant.

II. RELATIVE MOTION EQUATIONS ASSOCIATED WITH THE TWO FLIGHT CRAFT

2.1 Establishment of Coordinate Systems

Adopting the target orbit coordinate system 0- $\overline{ijk}$, $\vec{i}$ is along a radius vector direction. $\vec{j}$ is within the plane of orbit. And, $\vec{i}$ is perpendicular. Adopting the direction of motion as positive, $\vec{k}$ is perpendicular to planes of orbit and forms with $\vec{i}$ and $\vec{j}$ a right handed system.

Considering the orbits of the two flight craft coplanar, as a result, it is possible to make discussions within the coordinate system 0- $\overline{ij}$.

2.2 Relative Motion Equations for the Two Flight Craft

Assuming that the included angle between the radius vectors of the two flight craft is θ, the radius vector of

the tracking flight craft is r, and the radius vector of the target flight craft is rm, then, the relative positions of the two flight craft at any instant can be expressed as

(1)
$$\begin{cases} x = r\cos\theta - r_m \\ y = r\sin\theta \end{cases}$$

The relative speeds of the two flight craft can be expressed as

(2)
$$\begin{cases} V_x = \dot{r}\cos\theta - (r\sin\theta)\dot{\theta} - \dot{r}_m \\ V_y = \dot{r}\sin\theta + (r\cos\theta)\dot{\theta} \end{cases}$$

In this,

$$\dot{r} = \frac{\mu}{h} e\sin f \quad ; \quad \dot{r}_m = \frac{\mu}{h_m} e_m \sin f_m ; \quad \dot{\theta} = \dot{f} - \dot{f}_m = \frac{h}{r^2} - \frac{h_m}{r_m^2}$$

Equation (2) can also be written as

(3)

$$\begin{cases} V_x = \frac{\mu}{h} e\sin f\cos\theta - \left(\frac{h}{r^2} - \frac{h_m}{r_m^2}\right) r\sin\theta - \frac{\mu}{h_m} e_m \sin f_m \\ V_y = \frac{\mu}{h} e\sin f\sin\theta + \left(\frac{h}{r^2} - \frac{h_m}{r_m^2}\right) r\cos\theta \end{cases}$$

2.3 Relative Positions and Relative Velocities of the Two Flight Craft at Various Impulse Effect Points

Due to the fact that tracking flight craft orbits must all change at various impulse effect points, as a result, the relative statuses of the two flight craft must be given at various impulse points at instants before and after impulse effects.

1) Tracking Flight Craft in Initial Orbit at the First Impulse Point (In Orbit I)

At this time, the relative statuses of the two flight craft can be expressed as

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(4)
$$\begin{cases} x_{11} = r_1\cos\theta_1 - r_{m1} \\ y_{11} = r_1\sin\theta_1 \\ V_{x11} = \frac{\mu}{h_1} e_1 \sin f_1 \cos\theta_1 - \left(\frac{h_1}{r_1^2} - \frac{h_m}{r_{m1}^2}\right) r_1 \sin\theta_1 - \frac{\mu}{h_m} e_m \sin f_{m1} \\ V_{y11} = \frac{\mu}{h_1} e_1 \sin f_1 \sin\theta_1 + \left(\frac{h_1}{r_1^2} - \frac{h_m}{r_{m1}^2}\right) r_1 \cos\theta_1 \end{cases}$$

In these, the included angle between the radius vectors of the two flight craft $\theta 1$ can be expressed as

(5)
$$\theta_1 = f_1 - (f_{m1} + \alpha_m)$$

2) Tracking Flight Craft in Orbit II at the First Impulse Point

At this time, the relative positions of the two flight craft are

(6)
$$\begin{cases} x_{21} = r_1\cos\theta_1 - r_{m1} = x_{11}; \\ y_{21} = r_1\sin\theta_1 = y_{11} \end{cases}$$

The relative velocities of the two flight craft are

(7)
$$\begin{cases} V_{x21} = \frac{\mu}{h_2} e_2 \sin f_{21} \cos\theta_1 - \left(\frac{h_2}{r_1^2} - \frac{h_m}{r_{m1}^2}\right) r_1 \sin\theta_1 - \frac{\mu}{h_m} e_m \sin f_{m1} \\ V_{y21} = \frac{\mu}{h_2} e_2 \sin f_{21} \sin\theta_1 + \left(\frac{h_2}{r_1^2} - \frac{h_m}{r_{m1}^2}\right) r_1 \cos\theta_1 \end{cases}$$

Assuming that thrusts along horizontal directions impacting on tracking flight craft are ΔV1 and ΔV2 , the projections on the two coordinate axes are, respectively, ΔV_{x1}, ΔV_{y1}. One then has

(8)
$$\begin{cases} \Delta V_{x1} = \left(\frac{h_1}{r_1} - \frac{h_2}{r_1}\right) \sin\theta_1 \\ \Delta V_{y1} = \left(\frac{h_2}{r_1} - \frac{h_1}{r_1}\right) \cos\theta_1 \end{cases}$$

Speaking in terms of tracking flight craft, the relationships set out below are established at instants before and after thrust effects:

(9) $$h_2 = h_1 + r_1 \Delta V_1$$

Included angles between the two flight craft radius vectors can also be expressed as

(10) $$\theta_1 = f_{21} + \alpha_1 - (f_{m1} + \alpha_m)$$

3) The Second Impulse Point for Tracking Flight Craft in Orbit II

At this time, the relative motion states of the two flight craft can be expressed as /5

$$\begin{cases} x_{22} = r_2 \cos\theta_2 - r_{m2} \\ y_{22} = r_2 \sin\theta_2 \\ V_{x22} = \frac{\mu}{h_2} e_2 \sin f_{22} \cos\theta_2 - \left(\frac{h_2}{r_2^2} - \frac{h_m}{r_{m2}^2}\right) r_2 \sin\theta_2 - \frac{\mu}{h_m} e_m \sin f_{m2} \\ V_{y22} = \frac{\mu}{h_2} e_2 \sin f_{22} \sin\theta_2 + \left(\frac{h_2}{r_2^2} - \frac{h_m}{r_{m2}^2}\right) r_2 \cos\theta_2 \end{cases}$$

(11)

4) The Second Impulse Point for Tracking Flight Craft in Orbit III

At the second impulse point, the thrust on the tracking flight craft producing effects along horizontal directions is ΔV2. The projections of ΔV2 on the two coordinate axes are ΔV_{x2}, ΔV_{y2}.

At this time, the relative positions of the two flight craft are

$$\begin{cases} x_{31} = r_2 \cos\theta_2 - r_{m2} = x_{22} \\ y_{31} = r_2 \sin\theta_2 = y_{22} \end{cases} \tag{12}$$

The relative speeds of the two flight craft are

(13)

$$\begin{cases} V_{x31} = \dfrac{\mu}{h_3} e_3 \sin f_{31} \cos\theta_2 - \left(\dfrac{h_3}{r_2^2} - \dfrac{h_m}{r_{m2}^2}\right) r_2 \sin\theta_2 - \dfrac{\mu}{h_m} e_m \sin f_{m2} \\ y_{31} = \dfrac{\mu}{h_3} e_3 \sin f_{31} \sin\theta_2 + \left(\dfrac{h_3}{r_2^2} - \dfrac{h_m}{r_{m2}^2}\right) r_2 \cos\theta_2 \end{cases}$$

Moreover,

(14)

$$\begin{cases} \Delta V_{x2} = \left(\dfrac{h_2}{r_2} - \dfrac{h_3}{r_2}\right) \sin\theta_2 \\ \Delta V_{y2} = \left(\dfrac{h_3}{r_2} - \dfrac{h_2}{r_2}\right) \cos\theta_2 \end{cases}$$

Included angles between the radius vectors of the two flight craft can be expressed as

$$\theta_2 = f_{31} + \alpha_2 - (f_{m2} + \alpha_m)$$

(15)

5) The Third Impulse Point

At this time, the relative distances of the two flight craft have already passed to zero. This impulse is only to change the speed of the tracking flight craft, making it and the speed of the target flight craft the same.

The expression for the relative positions of the two flight craft is

(16) $$\begin{cases} x_{32} = r_3\cos\theta_3 - r_{m3} = 0 \\ y_{32} = r_3\sin\theta_3 = 0 \end{cases}$$

As a result, one has $\theta_3=0;\ r_3=r_{m3}$

At the third impulse point, at the instant before impulse thrust effects, relative speeds of the two flight craft can be expressed as /6

(17) $$\begin{cases} V_{x32} = \frac{\mu}{h_3}e_3\sin f_{32}\cos\theta_3 - \left(\frac{h_3}{r_3^2} - \frac{h_m}{r_{m3}^2}\right)r_3\sin\theta_3 - \frac{\mu}{h_m}e_m\sin f_{m3} \\ V_{y32} = \frac{\mu}{h_3}e_3\sin f_{32}\sin\theta_3 + \left(\frac{h_3}{r_3^2} - \frac{h_m}{r_{m3}^2}\right)r_3\cos\theta_3 \end{cases}$$

Taking $\theta_3=0$, $r_3=r_{m3}$ and substituting into the forms above, it is possible to obtain

(18)
$$\begin{cases} V_{x32} = \frac{\mu}{h_3} e_3 \sin f_{32} - \frac{\mu}{h_m} e_m \sin f_{m3} \\ V_{y32} = \frac{h_3}{r_{m3}} - \frac{h_m}{r_{m3}} \end{cases}$$

Thrusts acting along horizontal directions on tracking flight craft are ΔV3. The projections of ΔV3 on the two coordinate axes are respectively ΔV_{x3}, ΔV_{y3} Due to the fact that, at this time, the relative distances of the two flight craft has already passed through to zero, therefore, ΔVx3=0. Because of this, one has

(19)
$$\begin{cases} V_{x32} + \Delta V_{x3} = V_{x32} = 0 \\ V_{y32} + \Delta V_{y3} = 0 \end{cases}$$

The included angles between the two flight craft are

$$\begin{aligned} \theta_3 &= f_{32} + \alpha_2 - (f_{m3} + \alpha_m) \\ &= f_1 + \Delta f - (f_{m1} + \Delta f_m + \alpha_m) = 0 \end{aligned}$$

(20)

2.4 Instants of Various Impulse Effects

Assume that the two flight craft begin from t=0 and go to the completion of rendezvous at t=tf . In this way, the first impulse is applied at time t=0. The third impulse is applied at time t=tf. It is assumed that the time of the application of the second impulse is tc. One then has

(21)

$$0-\tau_2=\sqrt{\frac{a_2^3}{\mu}}(E_{21}-e_2\sin E_{21})$$

$$t_c-\tau_2=\sqrt{\frac{a_2^3}{\mu}}(E_{22}-e_2\sin E_{22})$$

(22)

Assume that τ_3 is the instant when the tracking flight craft passes through perigee in orbit III. On orbit III, the time of the second impulse effect is $t=t_c$. The time of the third impulse effect is $t=t_f$. Because of this,

(23) $$t_c-\tau_3=\sqrt{\frac{a_3^3}{\mu}}(E_{31}-e_3\sin E_{31})$$

(24)

$$t_f - \tau_3 = \sqrt{\frac{a_3^3}{\mu}}\,(E_{32} - e_3 \sin E_{32})$$

As far as target flight craft movements in fixed orbit (orbit IV) are concerned, assuming that the instant when it passes perigee is τ_m , then, one has /7

(25)

(26)

$$0 - \tau_m = \sqrt{\frac{a_m^3}{\mu}}\,(E_{m1} - e_m \sin E_{m1})$$

$$t_c - \tau_m = \sqrt{\frac{a_m^3}{\mu}}\,(E_{m2} - e_m \sin E_{m2})$$

$$t_f - \tau_m = \sqrt{\frac{a_m^3}{\mu}}\,(E_{m3} - e_m \sin E_{m3})$$

(27)

III. RESOLVING THE PROBLEMS OF RENDEZVOUS

3.1 Initial Conditions and Terminal Conditions of Rendezvous

Initial Conditions

(1) Assume that target flight craft and tracking flight craft initial orbits are already known, that is, h_m, a_m, e_m and h_1, a_1, e_1, as well as αm are already known.

(2) Assume that, when rendezvous begins, the initial positions and the relative statuses of the two flight craft are already known, that is, when t=0, f_{m1}, E_{m1}, r_{m1}, f_1, E_1, r_1 as well as x_{11}, y_{11}, V_{x11}, V_{y11}, θ_1 are already known.

Terminal Conditions

Assume that the general time of rendezvous t=tf is already known and that the relative positions and the relative speeds have all become zero.

3.2 Relations Between Times of Impulse Effects and Flight Craft True Anomalies

Assume that the time going from the first impulse to the second impulse is tc . During this period of time, the radius vectors, true anomalies, and eccentric anomalies associated with target flight craft change respectively from r_{m1}, f_{m1}, E_{m1} to r_{m2}, f_{m2}, E_{m2}.

Combining equations (25) and (26), it is possible to obtain

(28) $$t_c = \sqrt{\frac{a_m^3}{\mu}}(E_{m2} - E_{m1} - e_m(\sin E_{m2} - \sin E_{m1}))$$

By the same reasoning, combining equation (25) and (27), it is possible to obtain

(29) $$t_f = \sqrt{\frac{a_m^3}{\mu}}(E_{m3} - E_{m1} - e_m(\sin E_{m3} - \sin E_{m1}))$$

Within the interval (0,tc), tracking flight craft move in orbit II. Movement time is tc. From equation (21) and equation (22), one obtains

(30) $$t_c = \sqrt{\frac{a_2^3}{\mu}}(E_{22} - E_{21} - e_2(\sin E_{22} - \sin E_{21}))$$

Within the interval ((t_c, t_f)), tracking flight craft move in orbit III. Movement time is tf-tc. From equations (23) and (24) one obtains /8

(31) $$t_f - t_c = \sqrt{\frac{a_3^3}{\mu}}(E_{32} - E_{31} - e_3(\sin E_{32} - \sin E_{31}))$$

From the relationship between true anomalies and eccentric anomalies, it is possible to obtain:

$$\begin{cases} \sin E_{21} = \dfrac{\sqrt{1-e_2^2}\,\sin f_{21}}{1+e_2\cos f_{21}} \\ \sin E_{22} = \dfrac{\sqrt{1-e_2^2}\,\sin f_{22}}{1+e_2\cos f_{22}} \end{cases}$$

Taking the two forms above and substituting into equation (30), one obtains

$$t_c = \sqrt{\frac{a_2^3}{\mu}}\left[arc\sin\left(\frac{\sqrt{1-e_2^2}\,\sin f_{22}}{1+e_2\cos f_{22}}\right) - arc\sin\left(\frac{\sqrt{1-e_2^2}\,\sin f_{21}}{1+e_2\cos f_{21}}\right) - e_2\left(\frac{\sqrt{1-e_2^2}\,\sin f_{22}}{1+e_2\cos f_{22}} - \frac{\sqrt{1-e_2^2}\,\sin f_{21}}{1+e_2\cos f_{21}}\right)\right]$$

(32)

By the same reasoning, one has

$$t_f - t_c = \sqrt{\frac{a_3^3}{\mu}}\left[arc\sin\left(\frac{\sqrt{1-e_3^2}\,\sin f_{32}}{1+e_3\cos f_{32}}\right) - arc\sin\left(\frac{\sqrt{1-e_3^2}\,\sin f_{31}}{1+e_3\cos f_{31}}\right) - e_3\left(\frac{\sqrt{1-e_3^2}\,\sin f_{32}}{1+e_3\cos f_{32}} - \frac{\sqrt{1-e_3^2}\,\sin f_{31}}{1+e_3\cos f_{31}}\right)\right] \tag{33}$$

$$\begin{aligned} t_f = {} & \sqrt{\frac{a_2^3}{\mu}}\left[arc\sin\left(\frac{\sqrt{1-e_2^2}\,\sin f_{22}}{1+e_2\cos f_{22}}\right) - arc\sin\left(\frac{\sqrt{1-e_2^2}\,\sin f_{21}}{1+e_2\cos f_{21}}\right) \right. \\ & \left. - e_2\left(\frac{\sqrt{1-e_2^2}\,\sin f_{22}}{1+e_2\cos f_{22}} - \frac{\sqrt{1-e_2^2}\,\sin f_{21}}{1+e_2\cos f_{21}}\right)\right] \\ & + \sqrt{\frac{a_3^3}{\mu}}\left[arc\sin\left(\frac{\sqrt{1-e_3^2}\,\sin f_{32}}{1+e_3\cos f_{32}}\right) - arc\sin\left(\frac{\sqrt{1-e_3^3}\,\sin f_{31}}{1+e_3\cos f_{31}}\right) \right. \\ & \left. - e_3\left(\frac{\sqrt{1-e_3^2}\,\sin f_{32}}{1+e_3\cos f_{32}} - \frac{\sqrt{1-e_3^2}\,\sin f_{31}}{1+e_3\cos f_{31}}\right)\right] \end{aligned} \tag{34}$$

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Equations (32) and (33) are the relationships between impulse effect times and tracking flight craft true anomalies. Equation (34) represents the relationship between overall rendezvous time and two transitional orbit parameters as well as angles turned through by tracking flight craft in transitional orbits.

Besides that, from initial and terminal conditions, one knows that e_m, a_m, E_{m1}, f_{m1}, f_1, α_m, as well as rendezvous times tf are already known. In this way, from equation (29), it is possible to make iterative substitutions through to E_{m3}. From this, it is possible to calculate f_{m3}. Moreover,

$$\Delta f_m = f_{m3} - f_{m1}$$

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As a result, it is possible to obtain Δfm . Taking Δfm and substituting into equation (20), it is possible to obtain Δf. In this way, we obtain--for the entire rendezvous process--the geocentric angles Δ fm which target flight craft turn through and the geocentric angles Δ f which tracking flight craft turn through.

3.3 Necessary Conditions for Rendezvous

At the point of the third impulse, the two flight craft complete rendezvous. At this time, the speed and position of tracking flight craft in all cases are the same as the target flight craft. From energy equations and angular momentum

equations, one obtains

$$\frac{\mu}{a_m} - \frac{\mu}{a_3} = \frac{h_3^2}{r_{m3}^2} - \frac{h_m^2}{r_{m3}^2}$$

(35)

As a result, one has

$$h_3^2\left(\frac{1}{r_{m3}^2} - \frac{1}{r_2^2}\right) = \frac{\mu}{a_m} - \frac{\mu}{a_1} + \frac{h_m^2}{r_{m3}^2} - \frac{h_1^2}{r_1^2} + \frac{h_2^2}{r_1^2} - \frac{h_2^2}{r_2^2} \qquad (36)$$

In this, r2 satisfies

$$\frac{1}{r_2} = \frac{\cos\Delta f_1}{r_1} + (1 - \cos\Delta f_1)\frac{\mu}{h_2^2} - \frac{\mu e_1}{h_1 h_2}\sin f_1 \sin\Delta f_1 \qquad (37)$$

Combining the results above, it is possible to obtain

$$\frac{\sin\Delta f_2}{\mu r_2} h_3^2 + \left[\frac{e_1}{h_1}\sin f_1 \cos\Delta f_1 \cos\Delta f_2 - \frac{e_m}{h_m}\sin f_{m3} + \left(\frac{h_2}{\mu r_1} - \frac{1}{h_2}\right)\sin\Delta f_1 \cos\Delta f_2\right] h_3 - \sin\Delta f_2 = 0 \qquad (38)$$

If r_2 in equation (36) and equation (38) is represented by

equation (37), in this way, equations (36) and (38) turn into equations relating to orbit II moment of momentum h2 and orbit III moment of momentum h3. Equations (36) and (38) are conditions which h2 and h3 must satisfy.

3.4 Solution of Rendezvous Equations

Through equations (36) and (38) which must be satisfied by moments of momentum h2 and h3 associated with orbit II and orbit II as well as equation (9) relating to the three impulses ΔV1, ΔV2, and ΔV3, and the two equations below:

(39)

$$h_3 = h_2 + r_2 \Delta V_2$$

$$\Delta V_3 = \Delta V_{y3} = \frac{h_m}{r_{m3}} - \frac{h_3}{r_{m3}}$$

(40)

and also making use of equations (33) and (34) relating to moments of impulse effects, we are capable of obtaining the three impulses ΔV1, ΔV2, and ΔV3, as well as the times of impulse effects.

Discussions below are divided into several types of situations:

Δf1 and Δf2 satisfy: Δf1 + Δf2 = Δf

1) $\Delta f2$ Δ $k\pi$, (k=1,1,2,...); and $\Delta f_1 \neq 2n\pi$, $(n=0, 1, 2, \cdots)$)

At this time, equation (38) can change to be /10

(41)
$$h_3^2 + \mu r_2 \left[\frac{e_1}{h_1} \sin f_1 \cos \Delta f_1 \operatorname{ctg} \Delta f_2 - \frac{e_m}{h_m} \sin f_{m3} \csc \Delta f_2 + \left(\frac{h_2}{\mu r_1} - \frac{1}{h_2} \right) \sin \Delta f_1 \operatorname{ctg} \Delta f_2 \right] h_3 - \mu r_2 = 0$$

Let

(42)
$$D_1 = \frac{e_1}{h_1} \sin f_1 \cos \Delta f_1 \operatorname{ctg} \Delta f_2 - \frac{e_m}{h_m} \sin f_{m3} \csc \Delta f_2$$

(43)
$$D_2 = \left(\frac{h_2}{\mu r_1} - \frac{1}{h_2} \right) \sin \Delta f_1 \, ctg \Delta f_2$$

Take equations (42) and (43) and substitute into (41), solving for h3. In conjunction with this, considering h3 > 0, one gets

$$h_3 = -\frac{1}{2}\mu r_2(D_1 + D_2) + \frac{1}{2}\sqrt{\mu^2 r_2^2(D_1 + D_2)^2 + 4\mu r_2} \tag{44}$$

In equation (36), let

$$D_3 = \frac{\mu}{a_m} - \frac{\mu}{a_1} + \frac{h_m^2}{r_{m3}^2} - \frac{h_1^2}{r_1^2} \tag{45}$$

Taking equation (44) and substituting into equation (36), it is possible to obtain

$$\begin{aligned} &\mu^2 r_2^2(D_1 + D_2)^2 + 2\mu r_2 - \mu r_2(D_1 + D_2)\sqrt{\mu^2 r_2^2(D_1 + D_2)^2 + 4\mu r_2} \\ &= \left[2D_3 + 2\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)h_2^2\right] / \left(\frac{1}{r_{m3}^2} - \frac{1}{r_2^2}\right) \end{aligned} \tag{46}$$

In this, D1, D2, and D3 are respectively given by equations (42), (43), and (45). r2 is given by equation (37). Equation (46) is an equation relating to h2. Specifying Δf2, it is then possible to make use of computers to solve for h2. Taking the h2 values obtained and substituting into equation (44), it is possible to obtain h3.

2) Δf2=(2k+1)π, (k=0,1,2,...); $\Delta f_1 \neq n\pi$, (n=0,1,2...)
At this time, one has

(47)
$$h_3^2\left(\frac{1}{r_{m3}}+\frac{1}{r_2}\right)=2\mu$$

Take Δf2=(2k+1)π and substitute into equation (38). Moreover, let

$$D_4=\frac{e_1}{h_1}\sin f_1 \operatorname{ctg}\Delta f_1+\frac{e_m}{h_m}\sin f_{m3}\csc\Delta f_1$$
(48)

Considering $h_3 \neq 0$, then one has

(49) $$h_2 = -\frac{1}{2}\mu r_1 D_4 + \frac{1}{2}\sqrt{\mu^2 r_1^2 D_4^2 + 4\mu r_1}$$

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From equations (47) and (49), it is possible to obtain h3 and h2.

3) $\Delta f_2 = (2k+1)\pi$, $(k=0, 1, 2, \cdots)$; $\Delta f_1 = (2n+1)\pi$, $(n=0, 1, 2, \cdots)$

At this time, equation (38) is simplified to be

(50) $$\frac{e_1}{h_1}\sin f_1 = \frac{e_m}{h_m}\sin f_{m3}$$

As a result, as long as tracking flight craft initial conditions and final rendezvous conditions for the two flight craft satisfy equation (50), it is then possible to see the appearance of this type of situation. At this time, one has

(51) $$h_2^2 = 2\mu / \left(\frac{1}{r_2} + \frac{1}{r_1}\right)$$

(52) $$h_3^2 = 2\mu / \left(\frac{1}{r_2} + \frac{1}{r_{m3}}\right)$$

In this type of situation, from equations (51) and (52), it is possible to obtain h2 and h3.

4) $\Delta f_2 = 2k\pi$ or $\Delta f_1 = 2k\pi$, (k=0,1,2,...). At this time, the problem can be reduced to being a second impulse rendezvous problem. Consideration of it will not be given here.

Above, several situations are delineated, and calculation fomulae for moments of momentum h2 and h3 associated with two transitional orbits are specified when Δf_2 is given. Because of this, the magnitude of the three horizontal impulses can be precisely specified as shown below:

(53)
$$\Delta V_1 = \frac{h_2}{r_1} - \frac{h_1}{r_1}$$

(54)
$$\Delta V_2 = \frac{h_3}{r_2} - \frac{h_2}{r_2}$$

(55)
$$\Delta V_3 = \frac{h_m}{r_{m3}} - \frac{h_3}{r_{m3}}$$

The times of effect for impulses are given by equations (33) and (34).

Above, calculation formulae are given to solve for the magnitudes and times of effect of the three horizontal impulses associated with fixed time rendezvous under the actions of three horizontal impulses. Calculation formulae are also given for various types of orbital parameters associated with transitional orbit shifts during rendezvous processes.

Finally, it should be pointed out that this section conducts discussions in situations specified under the initial conditions for the two flight craft. If we select appropriate launch times, and make tracking flight craft possess appropriate initial attitudes, then, it is possible to obtain optimized solutions for fixed time rendezvous under the effects of three horizontal impulses.

IV. CONCLUSIONS

This article--on the basis of flight craft relative motion nonlinear equations--studies fixed time rendezvous problems of two flight craft in general coplanar elliptical orbits under the effects of three horizontal impulses. First of all, we discuss the relative motion statuses of the two flight craft at various impulse points before and after impulse effects. Following that, we derived the necessary conditions for completing rendezvous. In conjunction with that, we give parameter calculation formulae for two transitional orbits. Finally, we give calculation methods for the magnitudes of three impulses as well as the times of the impulse effects.

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